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Robust Algorithms for the Calculation of Full-Wave Electromagnetic Solutions

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A powerful technique is described for optimizing electromagnetic codes that involve the boundary element method and nonlinear eigenvalue problems. It is specifically adapted to an algorithm that handles propagation in periodic structures and replaces the standard determinant search and linear equation solution subroutines. Through mechanical speedup and improved numerical convergence, the run time for large cases is reduced by an order of magnitude. Details of technique, the electromagnetic algorithm, and numerical studies are presented.

1. INTRODUCTION

The evaluation of electromagnetic wave propagation in the presence of conducting structures is often carried out by boundary element methods [1]. One usually wants to evaluate the dependence of the wave vectors of propagating modes on the wave frequency and to find the spatial distribution of conductor current corresponding to these modes. For periodic structures the computation consists mainly of evaluating the wave-vectors for which a generalized impedance matrix $\mathbf{Z}$ is singular and of finding the corresponding eigenmodes [2]. Except for very simple structures, the matrix $\mathbf{Z}$ is large and highly ill-conditioned, so that finding the relevant eigenvalues is a difficult task, setting severe limitations on the size and complexity of systems which can be reliably solved.

We propose a new numerical approach which overcomes these difficulties. Numerical tests carried out show that the method is both robust and considerably more efficient than existing techniques. The proposed algorithm is based on the specific spectral features of the impedance matrix $\mathbf{Z}$ and on the fact that only one or a few eigen-modes are needed. It relies on a preconditioning transformation [3] of $\mathbf{Z}$ into a matrix which has the same relevant spectral properties but is much better conditioned. We then apply a modified Arnoldi method [4] which focuses on evaluating the few relevant eigenvalues only, in the most efficient manner.

The paper is organized as follows: Section 2 describes how the application of boundary element method to the EM wave propagation leads to an eigenvalue problem for the impedance matrix. Section 3 discusses the spectral properties of $\mathbf{Z}$, exposes the difficulties of finding its eigenvalues by standard methods, and describes the basics of the proposed preconditioning approach. Section 4 presents a full description of the preconditioned Arnoldi procedures and their efficient embedding in Newton iterations for the determination of propagation constants. Section 5 presents numerical examples which compare the power of the proposed methods with conventional procedures. The paper concludes with a short summary and a suggestion for possible extensions.

2. PROBLEM FORMULATION

The formulation of wave propagation through a structure will be briefly outlined below. For a more complete description see [2, 5].

The geometry of an electronic circuit is defined in a unit cell Ω in $\mathbb{R}^3$ of dimensions $\{d_x, d_y, d_z\}$. The cell is periodically

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extended implying periodic boundary conditions in the x, y directions, while a radiation boundary condition is imposed in the z direction. These boundary conditions describe waves travelling along the periodic structure.

A part of the unit cell is occupied by conductors, while the rest of the volume is assumed to have uniform electromagnetic properties. Let us consider time harmonic currents:

$$\mathcal{J}(\mathbf{x}, t) = J(\mathbf{x})e^{j\omega t} \tag{1}$$

The electromagnetic field E induced by conductor currents J is represented by

$$E = \mathcal{L}J, \tag{2}$$

where $\mathcal{L}$ is an appropriate integral operator. Since Ohm's law holds on the conductor surfaces, we have an algebraic constraint there,

$$E_t = RJ, \tag{3}$$

where R is the resistance, and E_t is the tangential surface field.

Combining these equations yields

$$\mathcal{Z}J \doteq (\mathcal{L}_t - R)J = 0, \tag{4}$$

where $\mathcal{L}_t$ is the tangential part of $\mathcal{L}$. Guided by Floquet–Bloch theorems for waves on periodic structures, one looks for currents J of the form

$$J = J_p e^{j\tilde{\mathbf{k}}\cdot\mathbf{x}}, \tag{5}$$

where J_p is periodic in the x, y directions, and $\tilde{\mathbf{k}} \in \mathbb{C}^3$ is the wave vector of a traveling wave. The periodic currents are spanned by

$$\{J_{mn}e^{j\mathbf{k}_{mn}\cdot\mathbf{x}}\}, \tag{6}$$

where the wave vectors are

$$\mathbf{k}_{mn} = \left(\frac{2\pi m}{d_x}, \frac{2\pi n}{d_y}, k_z\right), \quad m, n \in \mathbb{Z}. \tag{7}$$

By Maxwell's equations the vertical component k_z is determined by the constraint:

$$\|\mathbf{k}_{mn} + \tilde{\mathbf{k}}\| = k^2 = (\omega/C)^2. \tag{8}$$

The space of currents should be further restricted since J_p must vanish outside the conductors. The space of functions of the form (5) which vanish outside the conductors will be denoted as $S_{\tilde{\mathbf{k}}}$. Let us define an operator

$$\hat{\mathcal{Z}}(\tilde{\mathbf{k}}) : S_{\tilde{\mathbf{k}}} \to S_{\tilde{\mathbf{k}}} \tag{9}$$

as the restriction of $\mathcal{Z}$ to $S_{\tilde{\mathbf{k}}}$.

To obtain the propagating modes one applies (4) to find $J \in S_{\tilde{\mathbf{k}}}$ such that

$$\hat{\mathcal{Z}}(\tilde{\mathbf{k}})J = 0. \tag{10}$$

A discrete approximation is constructed by the method of moments described by Spielman and Harrington [1, 6]. This method approximates the current J in a finite dimensional space B of "basis functions" and the field E in a finite dimensional space T of "test functions." The resulting discrete version of (10) reduces to the following:

Find $\tilde{\mathbf{k}} \in \mathbb{C}^3$ and $J \in B$ such that

$$\int_\Omega \mathbf{v} \cdot \hat{\mathcal{Z}}(\tilde{\mathbf{k}})J = 0 \tag{11}$$

for all $\mathbf{v} \in T$. The basis functions in our application are piecewise linear in the current direction and piecewise constant in the cross direction (rooftop functions). Testing is done by line integration. This discretization method is described in detail by Rubin [2]. The matrix resulting from (11) will be called Z in the sequel.

3. THE NONLINEAR EIGENVALUE PROBLEM

The discrete equation (11) has a nontrivial solution only if the matrix Z is singular. This leads to the nonlinear complex matrix eigenvalue problem:

$$|Z(\tilde{\mathbf{k}})| = 0. \tag{12}$$

The aim is to find a wave vector $\tilde{\mathbf{k}}$ for which the boundary element matrix Z is singular. A common way to solve (12) is by performing Newton iterations for zeroing the determinant $|Z(\tilde{\mathbf{k}})|$. This determinant can be computed by multiplying the pivots of the LU factors of $Z(\tilde{\mathbf{k}})$. An alternative approach is to compute the small eigenvalues of the matrix and to construct a Newton process which drives one of the eigenvalues to zero. An algorithm which computes part of the spectrum of matrices of similar kinds by Arnoldi's method was presented by Natarajan [7].

Either of these methods requires at least two matrix factorizations for each Newton iteration. Considering the cubic complexity of the factorization, these algorithms are extremely time consuming for nontrivial models.

Besides this high cost, such techniques often encounter numerical problems due to the adverse spectral structure of the operator Z, resulting from application of the boundary element method. Typically, there are a few small eigenvalues of $Z(\tilde{\mathbf{k}})$, which cross the origin during the evolution in $\tilde{\mathbf{k}}$ and hence can be driven to zero. These are masked by many small eigenvalues

(of order k^2, where k is the operating frequency). About half of the spectrum is large (of order N^2, where N is the number of grid points in one of the space directions). Properties of the spectrum are further discussed below in Section 4.1. The vast dynamic range of the complex spectrum causes precision loss and results in a small convergence domain of the Newton process. In other words, a very close starting guess for $\tilde{\mathbf{k}}$ is usually required.

In this work we suggest a preconditioned Arnoldi procedure for finding the eigenvalues of Z which can be zeroed. Instead of solving (12) we look for small eigenvalues of the expression $P^{-1}Z(\tilde{\mathbf{k}})$. It is clear that this new matrix is singular whenever $Z(\tilde{\mathbf{k}})$ is, and thus, it has the same solution $\tilde{\mathbf{k}}$. We show that an appropriate preconditioner P can be found for which the new spectrum is extremely "well conditioned." This transformation of the problem results in three important features: (a) only one matrix factorization is needed, (b) the sensitivity to initial guess is reduced drastically, (c) several solutions can be found with a single preconditioner.

3.1. *The Eigenvalue Search*

The nonlinear matrix eigenvalue problem (11) is solved by a search for a vector $\tilde{\mathbf{k}}$ such that

$$\lambda_{\min}(Z(\tilde{\mathbf{k}})) = 0, \tag{13}$$

where $\lambda_{\min}$ denotes an eigenvalue of Z which is not larger in magnitude than any other eigenvalue. For our specific application it is assumed that the vector $\tilde{\mathbf{k}}$ is in the $\mathbf{x}$ direction: $\tilde{\mathbf{k}} = kx\hat{\mathbf{a}}_x$. The following method can be applied.

ALGORITHM 3.1. Newton iterations for $\lambda_{\min}$.

1. Guess an approximate kx
2. Construct the matrix $Z(kx)$, and find its small eigenvalues $\{\lambda_i\}$
3. Choose a smallest eigenvalue λ_0, and compute its approximate Jacobian,

$$J = \frac{\partial \lambda_0}{\partial kx} \simeq \frac{\lambda(kx + \Delta) - \lambda(kx)}{\Delta} \tag{14}$$

   using a sufficiently small increment Δ.
4. Update kx by Newton's rule:

$$kx^+ = kx^- - \frac{\lambda_0}{\dot{\lambda}_0} \tag{15}$$

5. Repeat from 2 until $|\lambda_0|$ is less than a threshold.

Apart from mathematical subtleties, such as the definition of $\dot{\lambda}_0$ at the points where eigenvalues cross each other, this method requires spectral analyses of full matrices, which is normally a hard computational task. This step is particularly difficult due to the spectral structure of the Z matrices occurring in this application as seen from the following example.

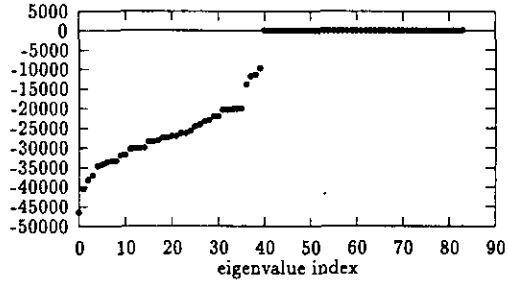

FIG. 1. The imaginary part of the spectrum of a 84 × 84 model matrix (the horizontal axis is the index of the eigenvalue).

The Z matrices are complex and non-Hermitian. Their eigenvalues may appear in any of the four quadrants of the complex plane. Figure 1 shows the imaginary part of the spectrum of a small (84 × 84) example. The real parts, which are not shown, are much smaller than the corresponding imaginary parts. The more detailed view of the eigenvalues with positive imaginary parts is shown in Fig. 2. There is an additional positive point at 48.7 which is not shown.

The structure of the spectrum which emerges from these figures thus consists of a group of eigenvalues, with large negative imaginary parts, and another part of the spectrum, which contains small eigenvalues with positive imaginary parts. The two parts are of similar orders.

This partition can be predicted by examining the analytic form of the operator $\mathcal{L}$ in (2) for two-dimensional structures. In the Fourier coordinates this operator can be written as [5]

$$\hat{\mathcal{L}}(\mathbf{k}) = \frac{1}{k_z}(k^2 I - \mathbf{k}\mathbf{k}^{\mathrm{T}}), \tag{16}$$

where $\mathbf{k}$ and k are defined by (7) and (8). The part $k^2 I/k_z$ of the operator is small and its imaginary part is positive. The part $-\mathbf{k}\mathbf{k}^T/k_z$ is of rank 1 (out of three space dimensions). Its imaginary part is negative and large, except for the mode

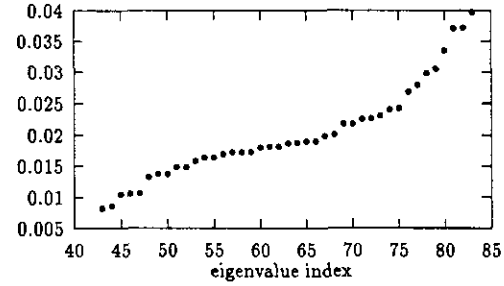

FIG. 2. The imaginary part of the eigenvalues with a positive imaginary part of the model matrix (expanded detail of Fig. 1).

corresponding to $m = n = 0$. This is the only mode which propagates at low frequency, while other modes may propagate at higher frequencies. The ill-conditioning between the large and the small part grows as k approaches zero, namely, at the quasi static limit.

The key observation for our approach is that, despite the adverse nature of the eigenvalues, the relative derivatives of most of them is nearly zero! As we modify the propagation constant kx, the relative variation of most of the eigenvalues is very small. Only a handful of them are "dynamic." These are precisely the ones which we are driving to zero.

It turns out that this property of a nearly stationary spectrum is very general and robust. For any two parameters $kx^{(1)}$, $kx^{(2)}$ which are not "too far apart," the preconditioned matrix

$$\tilde{Z} = Z(kx^{(2)})^{-1}Z(kx^{(1)}) \tag{17}$$

has many eigenvalues which are densely clustered around 1 and a handful of "dynamic" eigenvalues. These dynamic ones are the values which may cross the origin. Observe that $\tilde{Z}$ is singular whenever $Z(kx^{(1)})$ is singular, provided that $Z(kx^{(2)})$ is regular. This implies that a parameter kx for which $\tilde{Z}$ is singular also solves the original problem (13).

4. THE PROPOSED ALGORITHMS

Small and isolated parts of a matrix spectrum, such as the ones which have been described, are ideally suited for Arnoldi's method. The convergence of this algorithm was studied by Saad [8].

ALGORITHM 4.1. A preconditioned Arnoldi's procedure.

1. For an initial guess kx get a "nearby" number $kx^{(2)}$. (See the discussion below.)
2. Compute the matrix $Z(kx^{(2)})$ and its inverse.
3. Choose a normalized vector $\mathbf{u}_0$, and construct an orthonormal basis to the Krylov subspace generated by the matrix $Z(kx^{(2)})^{-1}Z(kx)$:
   - for $i = 1$ to m do
   - $\mathbf{v}_i = Z(kx^{(2)})^{-1}Z(kx)\mathbf{u}_{i-1}$
   - $\mathbf{u}_i = \mathbf{v}_i - \sum_{j=0}^{i-1}(\mathbf{v}_i, \mathbf{u}_j)\mathbf{u}_j$
   - $\mathbf{u}_i = \mathbf{u}_i/(\mathbf{u}_i, \mathbf{u}_i)$
4. Form the matrix:

$$B = U^*Z(kx^{(2)})^{-1}Z(kx)U, \tag{18}$$

   where $\{\mathbf{u}_i\}_0^m$ constitute the columns of U.
5. Compute the eigenvalues of B. The values which are not near 1 approximate the isolated eigenvalues of $Z(kx^{(2)})^{-1}Z(kx)$.
6. Compute the approximate eigenvectors by noting that they are columns of the matrix UM, where M stands for the matrix whose columns are the eigenvectors of B.

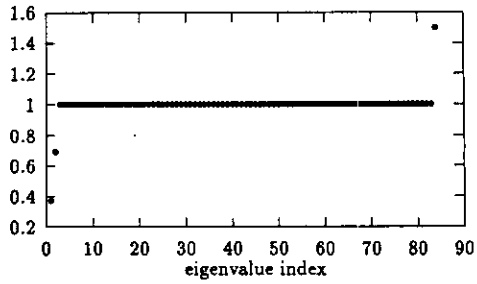

FIG. 3. The eigenvalues of the model matrix at $kx = 1.1k$, preconditioned by the matrix at $kx = 1.2k$.

Remarks. • The parameter $kx^{(2)}$ should be sufficiently close to kx, but the small eigenvalues of $Z(kx^{(2)})$ should be apart from the origin. It is not hard to satisfy these requirements by exploiting the physical meaning of the solution, which requires that it does not have a negative imaginary part (this would correspond to a "negative attenuation"). The insensitivity of the algorithm to the choice of $kx^{(2)}$ is a central factor in its robustness.

• The orthogonalization step (which is sometimes called modified Gram–Schmidt) is unnecessary in precise arithmetic. However, it is quite essential for numerical stability.

• The algorithm is insensitive to the choice of the initial vector $\mathbf{u}_0$. Strictly speaking, it must have nonzero components in the directions of the main eigenvectors. In practice, such components arise from numerical noise during the construction. We choose $\mathbf{u}_0$ to be the index set $\{1, 2, ..., n\}$.

• The accuracy of the approximate eigenvalue and the corresponding eigenvector is relative to the separation between the eigenvalues. We found that the choice $m = 10$ is adequate.

• The cost of the algorithm is expressed by the formula:

$$C = \tfrac{1}{3}n^3 + 2mn^2 + 0.5m^2n + m^3 + nm^2, \tag{19}$$

where n is the order of Z, and m is the number of Arnoldi iterations. The first term corresponds to the cost of the LU factorization of $Z(kx^{(2)})$. The second term represents the cost of a matrix multiplication and backward forward substitution, which are performed during each iteration. The third term is the cost of the orthogonalization. The fourth is the cost of computing the eigenstructure of the matrix **B**, and the last term represents the eigenvector computation (step 6). Therefore the cost for large matrices is $\sim\frac{1}{3}n^3 + 2mn^2$

Due to the robustness of this algorithm with respect to the preconditioning point $kx^{(2)}$, it is possible to use a common preconditioning matrix $Z(kx^{(2)})^{-1}$ for a whole Newton process. This results in the following method.

ALGORITHM 4.2. Newton–Arnoldi iterations for $\lambda_{\min}$.

1. Guess an approximate kx, and its perturbation $kx^{(2)}$
2. Compute the matrix $Z(kx^{(2)})$, and its inverse.
3. Compute the isolated eigenvalues of the matrix $Z(kx^{(2)})^{-1}Z(kx)$ by the preconditioned Arnoldi's method using Algorithm 4.1.
4. Choose one of the smallest eigenvalues $\lambda_0(kx)$.
5. Repeat steps 3 and 4 at a neighboring point to find $\lambda_0(kx + \Delta)$.
6. Compute the numerical derivative $\dot{\lambda}_0$ of λ_0

$$\dot{\lambda}_0(kx) = \frac{\lambda_0(kx + \Delta) - \lambda_0(kx)}{\Delta} \tag{20}$$

7. Update kx by Newton's rule:

$$kx^+ = kx^- - \frac{\lambda_0(kx^-)}{\dot{\lambda}_0(kx^-)} \tag{21}$$

8. Repeat from 2 until $|\lambda_0|$ is less than a threshold.

Remarks. • Instead of selecting a smallest eigenvalue, one may choose the eigenvalue with a minimal Newton update $|\lambda_0/\dot{\lambda}_0|$.

• The cost of the method is

$$C = \tfrac{1}{3}n^3 + (2l + 1)C_{\text{matgen}} + 4lmn^2 + o(n^2), \tag{22}$$

where l is the number of required Newton iterations, and C_{matgen} is the amount of computation required for constructing Z.

Note that the n^3 term in (22) is not multiplied by l. This means that the average cost of an iteration decreases as we keep using the same preconditioner. This fact is further exploited when several solutions are needed.

4.1. *Multiple Solutions*

The nonlinear eigenvalue problem usually has more than a single solution. Once the first solution we deal with is found, it is possible to start new Newton processes for other eigenvalues. It turns out that a single preconditioning matrix is often suitable for a whole group of solutions. In order to prevent the Algorithm 4.2 from converging back to the previous solutions, it is necessary to orthogonalize the Krylov subspaces constructed during the preconditioned Arnoldi iterations, with respect to the previously found left eigenvectors. The restriction of the search space to a subspace is sometimes called a "deflation" of the space.

Due to the nonlinearity of $Z(kx)$, the left eigenvectors at a specific kx are not biorthogonal with respect to the right eigenvectors at $kx + \varepsilon$. Only approximate orthogonality exists for a small ε. The deflation is thus merely a way to avoid the "attraction" of the previous solution. Once we arrive at a neighborhood of a new desired solution, the deflation should be stopped.

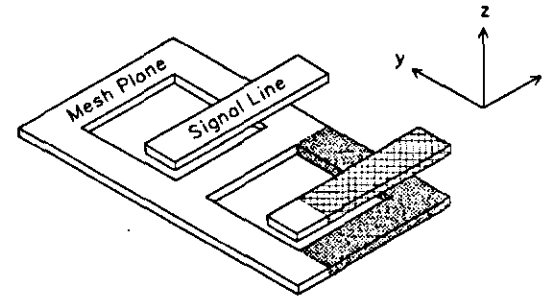

FIG. 4. The geometry of EX1. The unit cell is shaded.

The following algorithm computes a sequence of kx solutions and the corresponding eigenvectors.

ALGORITHM 4.3. Newton–Arnoldi iterations for multiple solutions.

1. Guess an approximate kx, and its perturbation $kx^{(2)}$
2. Compute the matrix $Z(kx^{(2)})$, and its inverse.
3. For $i = 1, 2, \ldots, l$ do:
   - Set a logical flag D = True, and a threshold T.
   - Perform an iteration of alg. 4.2. If D = True, then orthogonalize the Arnoldi vectors $\{\mathbf{u}_j\}$ with respect to the previously found solution vectors, before computing the eigenvalues of the Arnoldi matrix $\mathbf{B}$.
   - If the eigenvalue is smaller than T then set D = False.
   - Repeat from step 2 till convergence is reached.

Remarks. • As mentioned in the remarks following Algorithm 4.2, the average cost of an iteration decreases as the preconditioner is used several times. While a direct eigenvalue computation requires $O(n^3)$ operations for each matrix, our method needs a single LU factorization for the entire multiple solution search.

• The LU factors may be used for a whole sequence of neighboring problems. For example, if the frequency dependence on kx is required, then a single factorization is usually sufficient for a whole sequence of frequency points.

5. EXAMPLES

The algorithms presented here were compared to a more conventional method of applying Newton iterations to the determinant. This method will be denoted as "determinant iterations".

Figure 4 shows a simplified version of a signal-line structure used in some circuit boards and computer modules. A signal line is situated above a ground reference plane that is periodically perforated with apertures so that it becomes a mesh plane; the

TABLE I

Timing of EX1 with Various Discretization Grids

Grid	Degrees of freedom	Det. iter.		Algorithm 4.2	
		Iterations	Solution time	Iterations	Solution time
4 × 4 × 6	84	3	0.24	3	0.14
8 × 8 × 6	256	4	5.23	3	2.19
8 × 16 × 6	456	4	27.91	3	7.05
12 × 12 × 3	516	4	40.45	3	8.86
12 × 16 × 6	660	6	126.08	3	17.50

apertures allow for vertical interconnections. The solution is nearly TEM, except for the perturbations introduced by the apertures; the propagation constant kx, normalized to that of a TEM wave, is expected to be just above unity. One unit cell is shaded in the figure. The horizontal dimensions of the unit cell are 1 by 1 mm, and spacing between the mesh plane and the plane of the signal lines is 0.6 mm. This structure was modelled at a frequency of 0.4774 GHz. This structure is denoted as EX1.

Table I and Fig. 5 show the computing time required to reach convergence of the Newton iterations with different levels of discretization. These results represent the net solution time, while the time needed for the generation of the impedance matrices is ignored. The matrix generation code has been optimized by a table lookup organization, and by using FFT, so that the run time for large cases is governed by determinant computations, or by the Arnoldi iterations. Timing was done on an IBM RS/6000 workstation model 550.

The asymptotic complexity of both algorithms depends on the number of LU factorizations performed, since this is the only computation which has a cubic cost. Therefore, we expect the preconditioned algorithm to be faster than the determinant iterations by roughly twice the number of Newton iterations performed (the old method requires two matrix factorizations per iteration). In addition to this factor, Table I shows that the new method is more robust with respect to grid refinement. The number of iterations of the determinant method increases

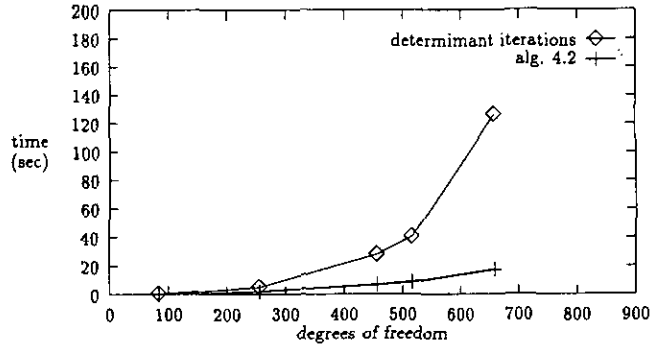

FIG. 5. Solution time for EX1, various discretizations.

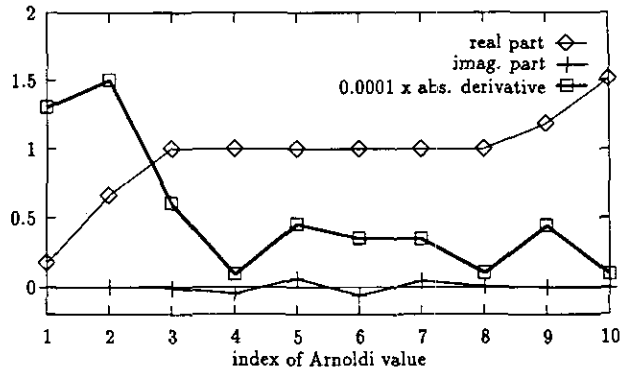

FIG. 6. The eigenvalues of EX1 computed by Algorithm 4.2 at $k_x = 1.05k$.

when the grid is refined, while the number of iterations needed by Algorithm 4.2 stays fixed. The solution time columns in Table I represent iteration times, excluding matrix generation time.

The actual structure of the eigenvalues computed by Algorithm 4.2 is shown in Fig. 6. This computation was done at the point $k_x = 1.05k$, while the solution is at $k_x = 1.0367k$. The significant features of the eigenvalue structure can be observed in this figure:

1. All of the eigenvalues, except for two small ones and two large ones are densely clustered around $k_x/k = 1$. Taking more Krylov vectors in the Arnoldi method would result in more and more eigenvalues in this cluster.

2. The imaginary part of the eigenvalues are small compared to the real part.

3. The smaller eigenvalues have a large derivative, compared with the rest of them.

4. The facts stated above lead to Newton updates which are much smaller for the first two eigenvalues than for the rest of them.

The second example consists of a signal-line sandwiched between two continuous ground planes, with a periodic array of orthogonal, non-touching signal lines running between the sig-

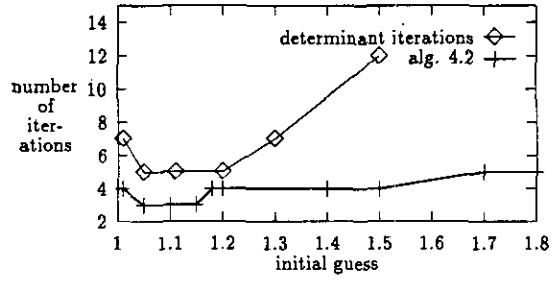

FIG. 7. Convergence region of the lossy structure (second example). A comparison of two algorithms.

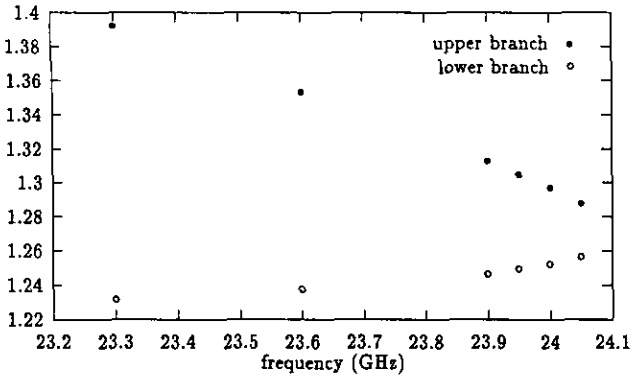

FIG. 8. Solutions of the waveguide example.

nal line and one of the ground planes. The conductors are lossy and, at the frequency of interest, significant dispersion exists.

Figure 7 shows a comparison of the number of iterations needed by determinant iterations and by Algorithm 4.2. The robustness of the preconditioned Arnoldi method with respect to the initial guess is seen clearly.

A third configuration is a rectangular dielectric waveguide with a periodic array of square holes, already considered in the literature [9]. (The electromagnetic algorithm also applies to dielectric structures by representing the polarization current through appropriate basis functions.) When it is solved by determinant iterations the numerical procedure faces difficulties around 24 GHz frequency.

Applying Algorithm 4.3 one immediately discovers that there are two solution branches, which cross near 24.1 GHz. This bifurcation point is the source of the numerical difficulties. The preconditioned eigenvalue analysis easily discovers all the "mobil" eigenvalues, which lead to potential solutions. This information is lacking in the determinant method. The two solution branches ae shown in Fig. 8.

The fourth structure is a section of a connector used to interconnect circuit boards in a computer [10]. Two outer ground pins are tied to a rectangular conducting bar with three signal pins in center. (These pins give rise to three low order modes). The structure is shown in Fig. 9.

TABLE II

Number of Iterations Required by Two Algorithms for Convergence to Three Solutions of Connector Example (Signifies No Convergence)

	Algorithm 4.3			Det. iter.		
Initial guess	1.12	1.13	1.41	1.12	1.13	1.41
1.02	—	—	—	—	—	—
1.07	4	3	—	7	—	—
1.10				6	—	—
1.12				4	—	—
1.124				4	—	—
1.128				—	—	—
1.25	3	4	—	—	8	—
1.35	6	3	4			
1.39				—	—	5
1.414				—	—	3
1.42	6	4	3	—	—	3
1.44				—	—	5
1.50	9	6	4	—	—	6
1.60	9	6	4	—	—	7
1.70	9	6	4	—	—	8
2.00	—	—	5	—	—	9

The number of iterations for this example, as solved by determinant iterations and by Algorithm 4.3 are shown in Fig. 11. This structure has 2160 degrees of freedom. The solutions are located roughly at 1.12, 1.13, 1.41. The determinant method converges slowly if the initial guess is not very close. The new method is far more forgiving with respect to inaccurate guesses. For a large interval of guesses all three solutions can be reached by a common preconditioner. Initial guesses which are below 1.3 and above 1.07 will trap the two lower solutions with a single preconditioner, but a higher starting point will be required for the solution at 1.41. The domains of convergence for the three solutions, when solved by Algorithm 4.3 are listed in Table II and displayed in Fig. 10.

Even when the initial guess is close enough for the determinant method, the new method is still much faster. We timed these solutions for very close guesses on an IBM 3090 computer with a vector facility. The total time required for the new

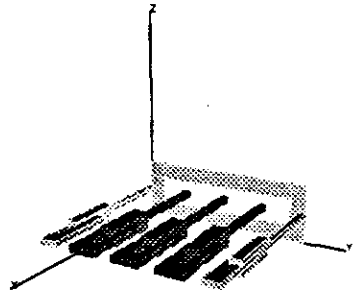

FIG. 9. A connector structure. The three inner pins are signal pins, and the two outer ones are ground pins. The shaded rectangle is a conducting bar.

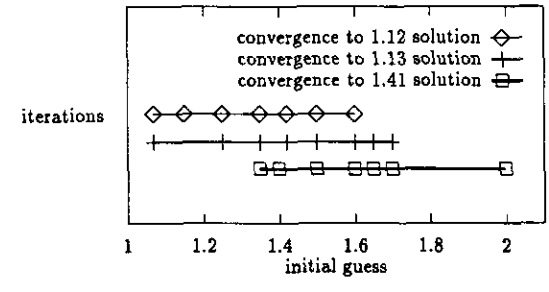

FIG. 10. Domain of convergence for connector example, using Algorithm 4.3. The x axis is the initial guess for kx. The marked points are successful runs.

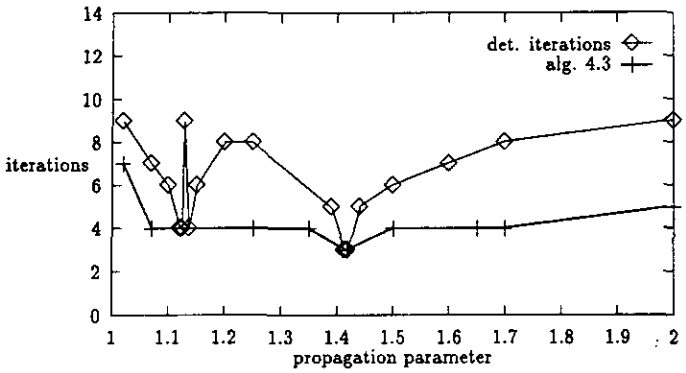

FIG. 11. Number of iterations needed for solution of connector example. Algorithm 4.3 is compared with the determinant method (det. iterations actually diverge at 1.128).

algorithm is 1246 s. The same solutions consumed 6240 s when solved by determinant iterations.

6. CONCLUSION

A new algorithm for reducing the run time in a certain class of electromagnetic code has been described and compared against a standard technique. Results calculated here, and from numerous other examples not shown, confirm both the speedup and improved convergence. Most importantly, the new technique works for every structure in the class. An important challenge for future work is the total elimination of the cubic complexity by alternative preconditioning techniques.

The methods described here may be extended to electromagnetic reflection problems, where there is a need to solve linear systems, rather than nonlinear eigenvalue problems.

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